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The concentration of stress and strain in finite thickness elastic plate containing a circular hole

Zheng Yang *, Chang-Boo Kim, Chongdu Cho, Hyeon Gyu Beom

Department of Mechanical Engineering, Inha University, Incheon 402-751, Republic of Korea

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Abstract

The elastic stress and strain fields of finite thickness large plate containing a hole are systematically investigated using 3D finite element method. It is found that the stress and strain concentration factors of the finite thickness plate are different even if the plate is in elasticity state except at notch root of plate surface. The maximum stress and strain do not always occur on the mid plane of plate. They occur on the mid plane only in thin plate. The maximum stress and strain concentration factors are not on mid plane and the locations of maximum stress and strain concentration factors are different in thick plate. The maximum stress and strain concentration factors of notch root increase from their plane stress value to their peak values, then decrease gradually with increasing thickness and tend to each constant related to Poisson's ratio of plate, respectively. The stress and strain concentration factors at notch root of plate surface are the same and are the monotonic descent functions of thickness. Their values decrease rapidly and tend to each constant related to Poisson's ratio with plate thickness increasing. The difference between maximum and surface value of stress concentration factor is a monotonic ascent function of thickness. The thicker the plate is or the larger the Poisson's ratio is, the larger the difference is. The corresponding difference of strain concentration factor is similar to the one of stress concentration factor. But the difference magnitude of stress concentration factor is larger than that of strain concentration factor in same plate.

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Keywords: Circular hole; Stress concentration factor; Strain concentration factor; Finite thickness

1. Introduction

Cracks in structures often initiate and propagate from the locations of stress or strain concentration. The stress and strain concentration locations are the critical structural details to determine the crack initiation and growth life of engineering structures. Despite careful detail-design, practically many structures contain stress and strain concentrations due to holes. Holes in structural components will create stress or strain concentrations and hence will reduce the mechanical properties. The majority of service cracks nucleate in the area of stress or strain concentration at the edge of a hole. Knowledge of stress and strain concentration in the vicinity

* Corresponding author. Tel.: +82 32 8607383; fax: +82 32 8681716.

E-mail address: zyang1964@hotmail.com (Z. Yang).

of a hole should be required for reliable design of structural components. Particularly, the increasing use of high strength materials in the design of structural parts with high mechanical performance requires a better understanding and modeling the behaviour of these structures. Exhaustive stress concentration factor figures and tables have been published by [Pilkey \(1997\)](#) and [Young \(Young and Budynas, 2002\)](#) which account for a wide variety of possible specimen configurations. However, many of these readily available sources of stress concentration factors consider only a two-dimensional theory of elasticity solution.

Experimental evidence shows that for relatively thin plates the crack either originates at the corner, where the hole meets the free surface of the plate, or at the center of the plate. On the other hand for relatively thick plates the crack almost always originates in the vicinity of the corner ([Broek, 1974](#)). Evidently, the stress and strain distributions near the edge of a hole are three-dimensional. Knowledge of the three-dimensional stress concentration factor is a prerequisite for an accurate design of structural components. The actual three-dimensional stress and strain fields near a curved boundary are very complex and there are only few analytical three-dimensional solutions available in the literature for non-trivial geometries and particular boundary conditions. By using series expansion and taking finite terms into account, [Sternberg et al. \(1949\)](#) obtained an approximate solution for 3D stress distributions near a circular hole in an infinite plate of arbitrary thickness. Detail analyses for the out-of-plane stress constraint were provided, but for stress concentrations only a brief discussion was given. To develop rigorous analytical solutions a number of approximate theories have been developed to consider the effects of three-dimensional constraint around a stress concentrator ([Gregory and Wan, 1988](#)). Many of these theories are based on an asymptotic expansion with respect to a small parameter, which is usually the ratio of the thickness to a characteristic length of the problem. However, underlying assumption limits the validity of any solutions obtained within these theories to only small values of the chosen parameter for thin plates. Analytical as well as numerical investigations reported in the literature show a slight difference between the in-plane stresses obtained from plane-strain theory and these 3D solutions. For example, the increase in the stress concentration factor for an infinite plate with a cylindrical hole subjected to uniaxial loading is less than 3% ([Sternberg et al., 1949](#)). In order to obtain a more general solution to the stress concentration problem, [Folias and Wang](#) developed a 3D solution using Navier's equation for plates of uniform thickness and with plate faces free of stress ([Folias and Wang, 1990](#)). Their results showed the stress concentration factor to be sensitive to the plate thickness and to Poisson's ratio. For thin plate, it was found that the stress concentration factor attains its maximum in the middle of the plate. On the other hand, for thick plate, the stress concentration factor attains its maximum close to the plate surfaces.

Recently, [Kotousov and Wang \(2002\)](#) presented analytical solutions for the three-dimensional stress distribution around typical stress concentrators in an isotropic plate of arbitrary thickness basing on the generalized plane strain theory assumption ([Kane and Mindlin, 1956](#)), which assumes that the out-of-plane strain is a constant in the thickness direction. The results were presented on the effects of the plate thickness and Poisson's ratio on the in-plane stress concentration factor and the out-of-plane stress constraint factor. It is shown that the stress concentration factor for a circular hole in an infinite plate is only slightly perturbed from the plane-strain solution over a wide range of thickness to radius ratio. Dealing with stress distributions due to holes, such hypothesis was rejected by [Krishnaswamy et al. \(1998\)](#). With reference to U and V-notches, [Li and Guo \(2001\)](#) also suggested that the assumption constant strain through the thickness can only be used in very thin or very thick plates and this kind of assumption was not suitable near the free surface. The validation of the assumption of generalized plane-strain theory in general stress concentration problems is suspectable. Considering plates of arbitrary thickness containing V-shaped notches, [Berto et al. \(2004\)](#) presented analytical solutions for the three dimensional stress field in the close neighborhood of the stress concentration region by combining [Kotousov and Wang's solution \(2002\)](#) for C_z and [Filippi et al.'s solution \(2002\)](#) for in-plane stresses. The influence of the plate thickness on three-dimensional stress field near notch root was examined by [Li et al. \(Li et al., 2000; Li and Guo, 2001\)](#) using 3D finite element analyses. While keeping the Poisson's ratio constant, [Li et al.](#) analyzed the plate thickness influence on the theoretical stress concentration factor, the stress distributions and the out-of-plane stress constraint factor. [Bellett et al. \(2005\)](#) showed experimentally that the common 2D methods for fatigue assessments of isotropic-notched bodies might lead to conservative predictions when applied to three-dimensional geometries. It has been confirmed that the SCF (Stress Concentration Factor) in the interior of the linear elastic isotropic plate with a hole or notch is significantly higher than that on the plate surface or the corresponding planar solutions ([Livieri and Nicoletto, 2003](#)). The

influence of Poisson's ratio on the thickness-dependent stress concentration factor (SCF) along the root of elliptic holes in elastic plates subjected to tension was investigated by use of three-dimensional finite element method. Some empirical formulae had been obtained by fitting the numerical results (She and Guo, 2007; Yu et al., 2007).

On the other hand, stress concentration factor is practically determined through strain measured on the plate surface. However the strain concentration factor is different from the stress concentration factor in real structures. This is a potential risk in 3D stress concentrated structures. So the strain concentration factor and the maximum strain concentration factor are also important parameters. The corresponding relationships between the interior stress concentration factor and the strain concentration factor for a 3D plate with a hole are invaluable for safety design or assessment of structures. Despite of its importance, to the best of authors' knowledge, there is not any report on the coupled influence of the Poisson's ratio and plate thickness upon the strain concentration factor and the relationship between stress and strain concentration factor of finite thickness plate.

In this paper, the coupled influence of the Poisson's ratio and plate thickness upon the stress concentration factor, the strain concentration factor and their relations of finite thickness plate containing a hole subjected to uniaxial tension are vastly investigated using the finite element method. The main purpose of this work is not primarily to provide numerical solutions for certain geometries but to highlight to the effect of Poisson's ratio on stress and strain concentration factors for the three-dimensional stress or strain concentration problems and consequently to provide general advice used to investigate the fatigue strength of materials.

2. Computational procedure and modeling

2.1. Definition

This study considers the finite thickness large plate containing a hole subjected to uniaxial tension (see Fig. 1). The radius of hole, the thickness, height and width of plate are a , $2B$, $2H$ and $2W$, respectively. The plate material is homogeneous, isotropic and elastic. The plane x – y (plane $z = 0$) is the mid plane of plate and the two plate surfaces are $z = B$ and $z = -B$, respectively. For convenience, the line corresponding z -axis is named as notch root in this paper. The stress concentration factor (K_σ) is

$$\begin{aligned} K_\sigma &= \sigma_{yy} / \sigma_{net}, \\ K_{\sigma 0} &= \sigma_{yy} / \sigma_{net} \quad \text{at } x = y = 0. \end{aligned} \quad (1)$$

The strain concentration factor (K_ε) is

$$\begin{aligned} K_\varepsilon &= \varepsilon_{yy} / \varepsilon_{net}, \\ K_{\varepsilon 0} &= \varepsilon_{yy} / \varepsilon_{net} \quad \text{at } x = y = 0. \end{aligned} \quad (2)$$

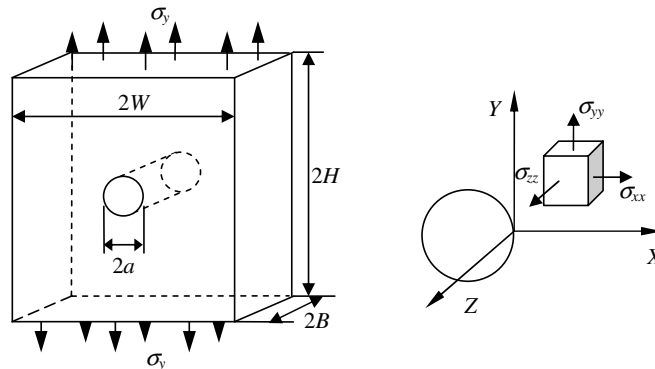


Fig. 1. The analysis model and the coordinate system.

Here, σ_{yy} and ε_{yy} are the opening stress and strain of plate, $\sigma_{net} = \sigma_y W / (W - a)$ and $\varepsilon_{net} = \sigma_y W / E(W - a)$ are the mean stress and strain of the net section on the ligament, respectively. E is the Young's modulus. σ_y is the remotely applied nominal stress. The $K_{\sigma 0}$ and $K_{\varepsilon 0}$ are K_σ and K_ε at the notch root, respectively. The maximum values of K_σ and K_ε along the notch root are denoted as $K_{\sigma \max}$ and $K_{\varepsilon \max}$, of which values on the mid plane of the plate are denoted as $K_{\sigma \text{ mp}}$ and $K_{\varepsilon \text{ mp}}$, respectively. K_0 is the stress concentration factor corresponding to the plate of plane stress state. According to the theory of elasticity (Timoshenko and Goodier, 1970), K_0 is independent of the Poisson's ratio and Young's modulus of plate.

For the stress fields in the finite thickness plate, two constraint parameters are introduced to describe the 3D characteristics of the stress fields near the notch root. The out-of-plane stress constraint factor is

$$T_z = \frac{\sigma_{zz}}{\sigma_{xx} + \sigma_{yy}}, \quad (3)$$

and the in-plane stress factor is

$$T_x = \frac{\sigma_{xx}}{\sigma_{yy}}. \quad (4)$$

$T_z = 0$ for the plane stress state and $T_z = \nu$ for the plane strain state. For the finite thickness plate of elastic material, $0 \leq T_z \leq \nu$, ν is the Poisson's ratio of material.

2.2. Finite element model

All analyses are carried out using the code ANSYS. Owing to the symmetry, only one-eighth of each plate is modeled. Appropriate boundary condition constraints are placed at all planes of symmetry. The large plate assumes that the width and height of plate may be 100 times of the hole diameter, respectively. In our 3D FE computations, the radius of hole is 1 mm, both the half-height and half-width of plate are set to be 100 mm. The plate thickness is varied from 0 to 5 mm. Poisson's ratio is from 0.15 to 0.45 and Young's modulus, E is set to be 200 GPa. A uniform stress of 100 MPa is applied on the boundary of the plate that is remote from the hole. To avoid human-made uncertainties in choosing the element types, the standard simple linear elements are applied. The finite element meshes are constructed with 8-node solid linear elements. 50 and 100 planar layers are divided through the half-thickness of the plate for thinner and thicker plate, respectively. To accommodate the variations of the field quantities through the plate thickness, the thickness of each successive element layer is gradually reduced toward the plate surface. Within each layer, the size of element decreases gradually with distance from the notch root decreasing. These 3D models represent a compromise between the required level of mesh refinement to solve the in-plane and through-thickness gradients of the stress fields and the extensive computation times required for each analysis.

3. In-plane stress distribution

The T_x distribution on different plane layer is very similar, so is the K_σ . They have relation to the thickness of plate and the location of plane layer. For plate with a hole, the in-plane stress ratio and stress concentration factor on different plane layers near the hole can be uniformly expressed, respectively as:

$$T_x\left(\frac{z}{B}\right) = h\left(\frac{x}{a}, \frac{z}{B}, \frac{B}{a}\right), \quad (5)$$

$$K_\sigma\left(\frac{z}{B}\right) = g\left(\frac{x}{a}, \frac{z}{B}, \frac{B}{a}\right). \quad (6)$$

Many efforts have been made to construct approximate expressions for stress distribution ahead of a notch-tip in elastic bodies (Creager and Paris, 1967; Glinka and Newport, 1987; Shin et al., 1994; Lazzarin and Tovo, 1996). The best-known expressions for calculating stresses in the vicinity of a notch-tip are for circular and elliptical notches in an infinite plate under remote loading. When the stress concentration factor of notch root, $K_{\sigma 0}$, is introduced, the stress components in the plane $y = 0$ in an infinite plate having a circular hole can be obtained under uniaxial loading condition from the classical solution (Timoshenko and Goodier, 1970):

$$\sigma_{xx} = \frac{K_{\sigma 0} \sigma_y}{2} \left(1 + \frac{x}{a}\right)^{-2} \left[1 - \left(1 + \frac{x}{a}\right)^{-2}\right], \quad (7)$$

$$\sigma_{yy} = \frac{K_{\sigma 0} \sigma_y}{3} \left[1 + \frac{1}{2} \left(1 + \frac{x}{a}\right)^{-2} + \frac{3}{2} \left(1 + \frac{x}{a}\right)^{-4}\right]. \quad (8)$$

It is easy to derive from Eqs. (7) and (8) that

$$T_x = \frac{\sigma_{xx}}{\sigma_{yy}} = \frac{3 \left(1 + \frac{x}{a}\right)^{-2} \left[1 - \left(1 + \frac{x}{a}\right)^{-2}\right]}{\left[2 + \left(1 + \frac{x}{a}\right)^{-2} + 3 \left(1 + \frac{x}{a}\right)^{-4}\right]}, \quad (9)$$

$$\frac{K_{\sigma}}{K_{\sigma 0}} = \frac{1}{3} \left[1 + \frac{1}{2} \left(1 + \frac{x}{a}\right)^{-2} + \frac{3}{2} \left(1 + \frac{x}{a}\right)^{-4}\right]. \quad (10)$$

Figs. 2 and 3 show the distributions of in-plane stress ratio and stress concentration factor in front of notch root on different plane layer for thickness $B/a = 4.0$ and $B/a = 1.0$, respectively. In Fig. 3, the stress concentration factor is normalized by its value at the notch root on the same plane layer. It can be easily seen that the curves $T_x \sim x/a$ and $K_{\sigma}/K_{\sigma 0} \sim x/a$ are different on different plane layer paralleling mid plane for both $B/a = 4.0$ and $B/a = 1.0$. The curves $T_x \sim x/a$ are sensitive to the location z/B of plane layer and the Eq. (9) can describe the T_x distribution well only near mid plane or near the notch root. The curves $K_{\sigma}/K_{\sigma 0} \sim x/a$ are insensitive to the location z/B of plane layer except in a very narrow region near the plate surface. But it should be noticed that the stress concentration factor along the notch root is not uniform and the stress concentration factor used in these expressions is the value of notch root corresponding to each z/B plane layer.

Comparisons of solutions (Eqs. (9) and (10)) with 3D results, the in-plane stress ratio and stress concentration factor distribution on the mid plane ($z/B = 0$ plane) can be predicted very well by 2D solution modified by considering 3D effect, the distributions of in-plane stress ratio and stress concentration factor in the vicinity of notch root can also be predicted using the modified 2D solutions except in narrow region near the plane surface.

4. The distributions of stress and strain concentration factor near the notch root

4.1. The relation of stress, strain concentration factor and plate thickness

In the same plate, the through-thickness distribution of stress concentration factor is different from the one of strain concentration factor. The maximum stress and strain do not always occur on the mid plane of the

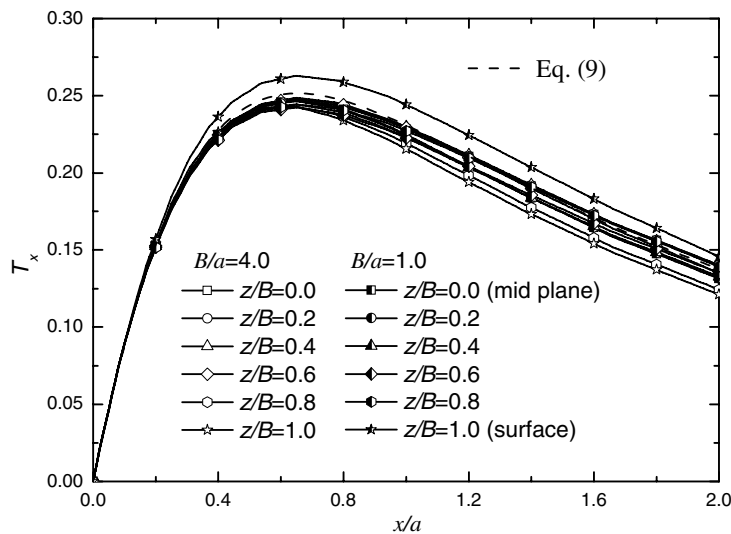


Fig. 2. The distribution of in-plane stress ratio in front of notch root on different plane layer for thickness $B/a = 4.0$ and $B/a = 1.0$.

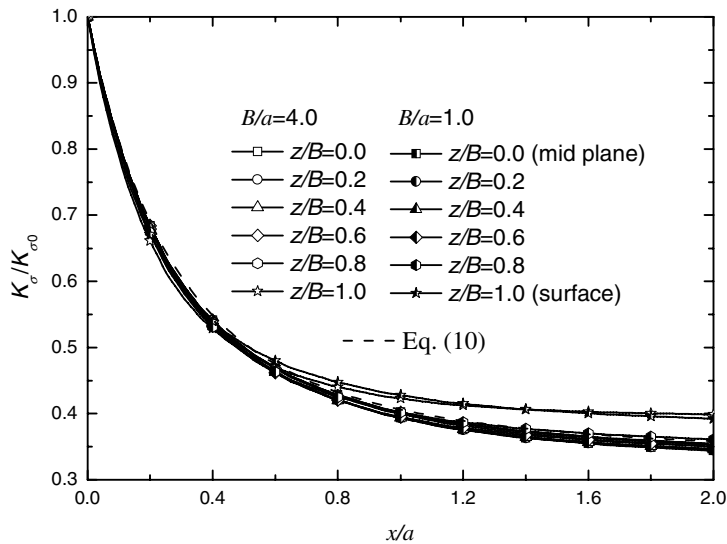


Fig. 3. The distribution of normalized stress concentration factor in front of notch root on different plane layer for thickness $B/a = 4.0$ and $B/a = 1.0$.

plate, shown in Fig. 4, in which the stress and strain concentration factors are normalized by each value of mid plane. The maximum values of K_σ and K_ϵ occur on the mid plane only in thin plates. The variations of stress and strain concentration factor with z/B are weak in the middle region, and the greater the dimensionless thickness (B/a) is, the more stable the stress and strain concentration factor become in this region. When the thickness of the plate exceeds a specific thickness (B_σ^*) that is called the transition thickness of stress concentration factor, the location of the maximum value of K_σ shifts gradually from mid plane to plate surface with the thickness increasing. When the thickness of the plate exceeds a specific thickness (B_ϵ^*) that is the called transition thickness of strain concentration factor, the location of the maximum value of K_ϵ shifts gradually from mid plane to plate surface with the thickness increasing. The thicker the plate is, the larger the K_σ and K_ϵ differences between the maximum value and the value of mid plane or surface are. There is a strong disturbed region near the plate surface. It is obvious that these locations of maximum K_σ and K_ϵ are different in the same plate, so the transition thickness of stress concentration factor is different from the one of strain concentration factor. In general, the locations of maximum K_ϵ are closer to plate surface than that of maximum K_σ in thick plate.

For different Poisson's ratio, the variations of stress concentration factor (K_σ) and strain concentration factor (K_ϵ) at the notch of mid plane ($K_{\sigma \text{ mp}}$, and $K_{\epsilon \text{ mp}}$), maximum value ($K_{\sigma \text{ max}}$, and $K_{\epsilon \text{ max}}$) and plate surface ($K_{\sigma \text{ sur}}$, and $K_{\epsilon \text{ sur}}$) versus the normalized thickness (B/a) are plotted in Fig. 5, where K_σ and K_ϵ are normalized by the value of plane stress state (K_0). For the plane stress state of elasticity solid, the values of K_σ and K_ϵ at the notch root are the same and independent of Poisson's ratio. The stress concentration factor of plane stress state ($B/a = 0$) is equal to the one of plane strain state ($B/a = \text{infinity}$). So there is an interesting phenomenon that the stress concentration factor at the notch root of mid plane, $K_{\sigma \text{ mp}}$, in a finite thickness plate is not a monotonic function of thickness. The $K_{\sigma \text{ mp}}$ increases from its plane stress value K_0 at $B/a = 0$ to its peak value, $K_{\sigma \text{ mpmax}}$, at about $B/a = 1.2$, then decreases gradually with increasing B/a and tends to its plane strain value when B/a becomes large enough. The values of $K_{\sigma \text{ mp}}/K_0$ increase with the Poisson's ratio and the $K_{\sigma \text{ mpmax}}/K_0$ value of the plate with Poisson's ratio 0.4 is about 1.07. The larger the Poisson's ratio is, the larger the value of $K_{\sigma \text{ mpmax}}/K_0$ is.

It is important to observe that the $K_{\sigma \text{ mp}}/K_0 - B/a$ and $K_{\epsilon \text{ mp}}/K_0 - B/a$ curves are different. The $K_{\epsilon \text{ mp}}/K_0 - B/a$ curve is similar to the $K_{\sigma \text{ mp}}/K_0 - B/a$ curve and also not a monotonic function of thickness. The $K_{\epsilon \text{ mp}}$ increases from its plane stress value K_0 at $B/a = 0$ to its peak value, $K_{\epsilon \text{ mpmax}}$, at about $B/a = 0.7$, but then decreases gradually with increasing B/a and tends to a value which is lower than its value of plane stress state when B/a becomes large enough. The values of $K_{\epsilon \text{ mp}}/K_0$ increase with the Poisson's ratio and the

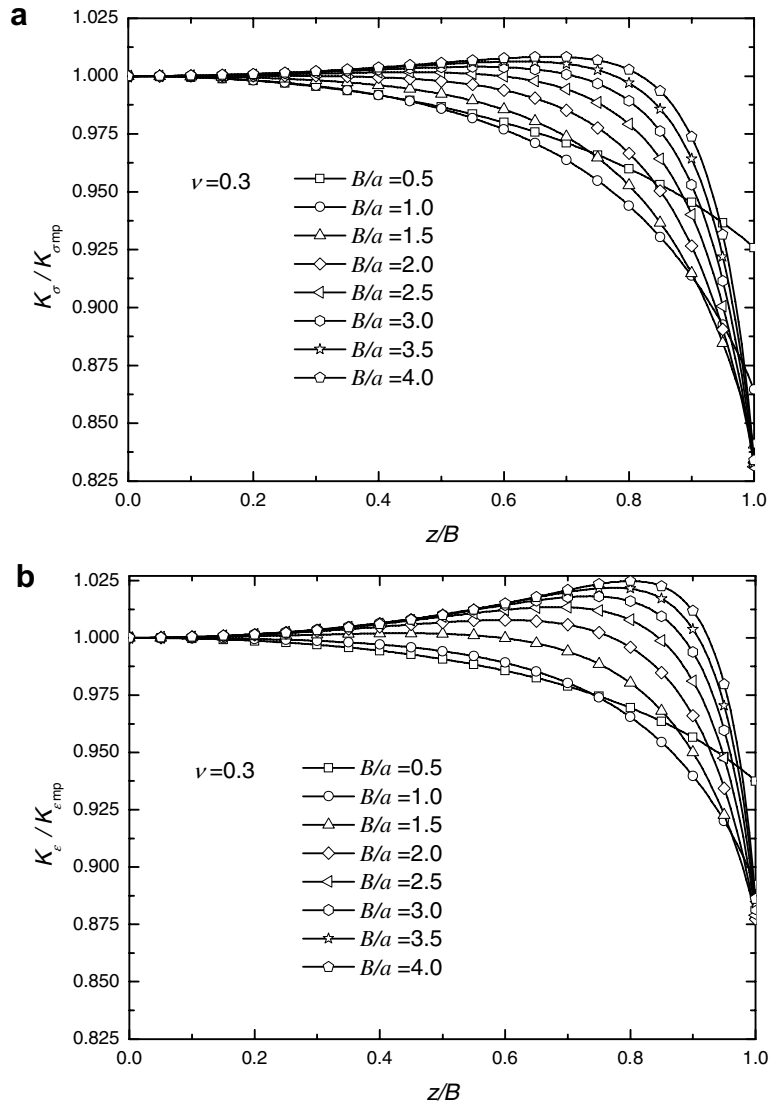


Fig. 4. The through-thickness distributions of the (a) stress and, (b) strain concentration factor along notch root for different thickness.

$K_{\varepsilon mpmax}/K_0$ value of the plate with Poisson's ratio 0.4 is about 1.02. Though the $K_{\varepsilon mpmax}/K_0$ value is lower than the $K_{\sigma mpmax}/K_0$ value, the $K_{\varepsilon mp}$ will decrease to the value below its plane stress value. When $B/a = 5$, the $K_{\varepsilon mp}/K_0$ value of the plate with Poisson's ratio 0.4 is only about 0.91 and it is about 10% lower than the $K_{\sigma mp}/K_0$ value of the same plate. The larger the Poisson's ratio of plate is, the larger the value of $K_{\varepsilon mpmax}/K_0$ is and the lower the $K_{\varepsilon mp}/K_0 - B/a$ curve tends to when B/a becomes large enough. The lower limits of $K_{\varepsilon mp}/K_0 - B/a$ curve for the plate with Poisson's ratio 0.2, 0.3 and 0.4 are about 0.98, 0.94 and 0.90, respectively. The transition points of maximum K_σ/K_0 and K_ε/K_0 ($K_{\sigma max}/K_0$ and $K_{\varepsilon max}/K_0$) are all in descending segment of the curves. The $K_{\sigma max}/K_0$ and $K_{\varepsilon max}/K_0$ decrease gradually with increasing B/a and tend to each constant for different Poisson's ratio when B/a becomes large enough. This constant of stress concentration factor will be larger than 1. The larger the Poisson's ratio of plate is, the larger the constant of stress concentration factor tends to when B/a becomes large enough, whereas this constant of strain concentration factor will be smaller than 1. The larger the Poisson's ratio is, the smaller this constant of strain concentration factor tends to when B/a becomes large enough.

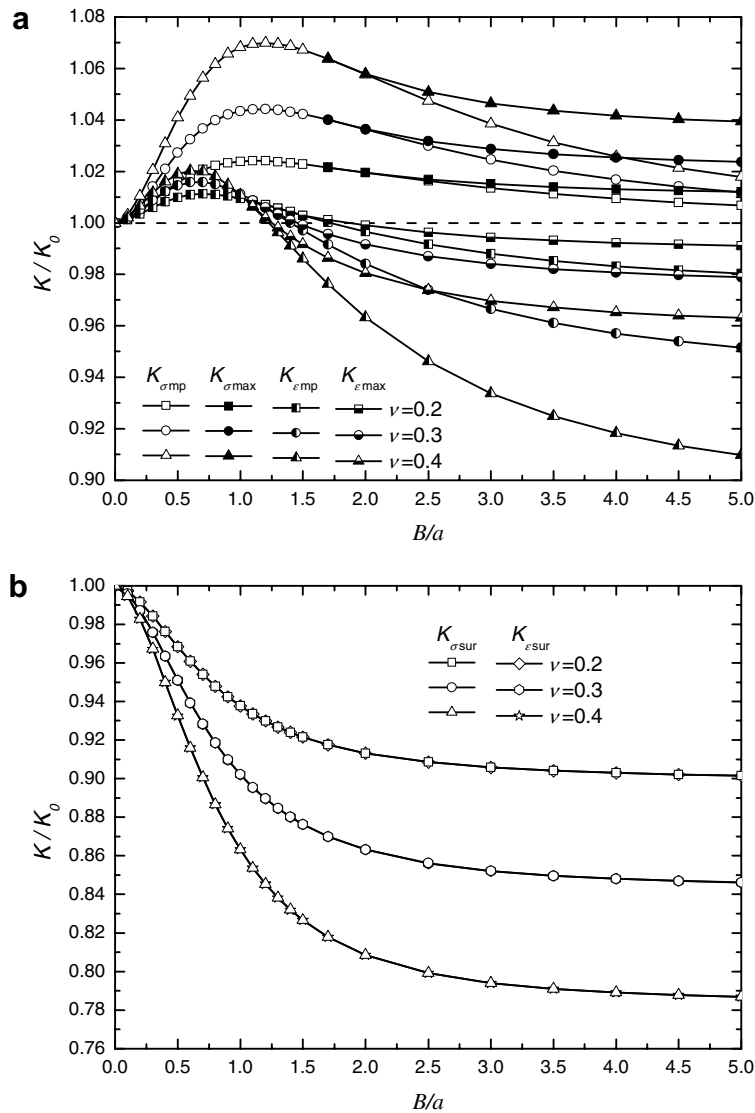


Fig. 5. The stress and strain concentration factors at the notch root of (a) mid plane, maximum and (b) plate surface versus normalized thickness for different Poisson's ratio.

The values of $K_{\sigma sur}$ and $K_{\epsilon sur}$ at the notch root of plate surface are the same for all different thickness with each Poisson's ratio from the FE results. So the $K_{\sigma sur}$ and $K_{\epsilon sur}$ are denoted by one term K_{sur} . The value of K_{sur} decreases rapidly and becomes lower than the corresponding plane stress or plane strain values. The $K_{sur}/K_0 - B/a$ curves are the monotonic descent functions of thickness that are different from the $K_{\sigma mp}/K_0 - B/a$ curves. The value of K_{sur} decreases from its plane stress value K_0 at $B/a = 0$ tends to each lower limit value for different Poisson's ratio when B/a becomes large enough. All of the K_{sur} for different thickness are smaller than their values of plane stress and the mid plane corresponding to each thickness and Poisson's ratio. The larger the Poisson's ratio is, the higher the decreasing rate of $K_{sur}/K_0 - B/a$ curves is, the lower the $K_{sur}/K_0 - B/a$ curves tend to when B/a becomes large enough. The K_{sur} of the plate with Poisson's ratio 0.4 is about 22% lower than the value corresponding plane stress state when the dimensionless thickness B/a is larger than 5.

The Poisson's ratio cannot change the distribution patterns of curve $K_{\sigma}K_{\sigma mp} - z/B$ and $K_{\epsilon}K_{\epsilon mp} - z/B$ of the same thickness plate and only change the difference degree between the mid plane value and the maximum value along thickness, or the surface value of stress and strain concentration factor, shown in Fig. 6. The larger

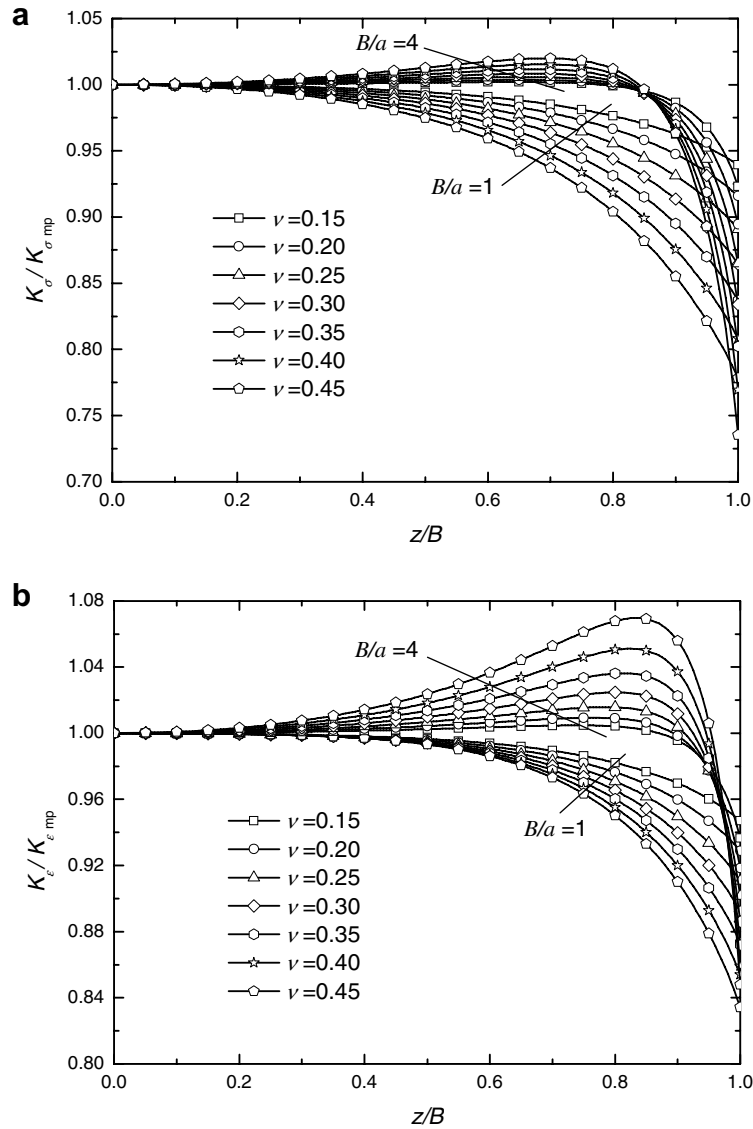


Fig. 6. The through-thickness distributions of (a) the stress and, (b) strain concentration factor along notch root for different Poisson ratio.

the Poisson's ratio is or the thicker the plate is, the larger this difference degree is. When the Poisson's ratio changes from 0.15 to 0.45, for the plate of thickness $B/a = 1$ (thin plate), the difference between mid plane value and surface value of stress concentration factor changes from 6.1% to 22.2% and the corresponding difference of strain concentration factor changes from 5.3% to 16.6%, respectively, whereas for the plate of thickness $B/a = 4$ (thick plate), the difference of stress concentration factor changes from 7.9% to 28.5% and the difference of strain concentration factor changes from 6.8% to 22.2%, respectively.

4.2. The difference between surface and maximum value of stress and strain concentration factor

The difference between mid plan and surface value of stress concentration factor is not a monotonic function of thickness. After its maximum value, it decreases gradually with increasing B/a and tends to a constant value corresponding different Poisson's ratio when B/a becomes large enough. But the difference between

maximum and surface value of stress concentration factor is a monotonic ascent function of thickness. It increases with increasing B/a and rapidly tends to other constant corresponding each Poisson's ratio when B/a becomes large enough. This difference relationship of strain concentration factor is similar to the one of stress concentration factor, shown in Fig. 7. The constant of stress concentration factor between maximum and surface value may be over 11.1%, 17.8% and 25.3% for plate with Poisson's ratio 0.2, 0.3 and 0.4, respectively. This constant of strain concentration factor may be over 9.0%, 13.4% and 17.6% for plate with Poisson's ratio 0.2, 0.3 and 0.4, respectively.

The through-thickness distributions of $K_\sigma/K_{\sigma \text{ mp}}$ and $K_\varepsilon/K_{\varepsilon \text{ mp}}$ are sensitive to the thickness and Poisson's ratio. The effects of thickness and Poisson's ratio on the through-thickness distributions of $K_\sigma/K_{\sigma \text{ mp}}$ and $K_\varepsilon/K_{\varepsilon \text{ mp}}$ are shown in Figs. 4 and 6, respectively. The K_σ and K_ε at the notch root of plate surface decrease as thickness or Poisson's ratio increases. It is also shown that the K_σ on mid plane in a finite thickness plate is higher than that in the plane stress or plane strain states and the K_ε on mid plane in a finite thickness plate may be lower than that in the plane stress or plane strain states. For the thick plate, the maximum K_σ which is larger

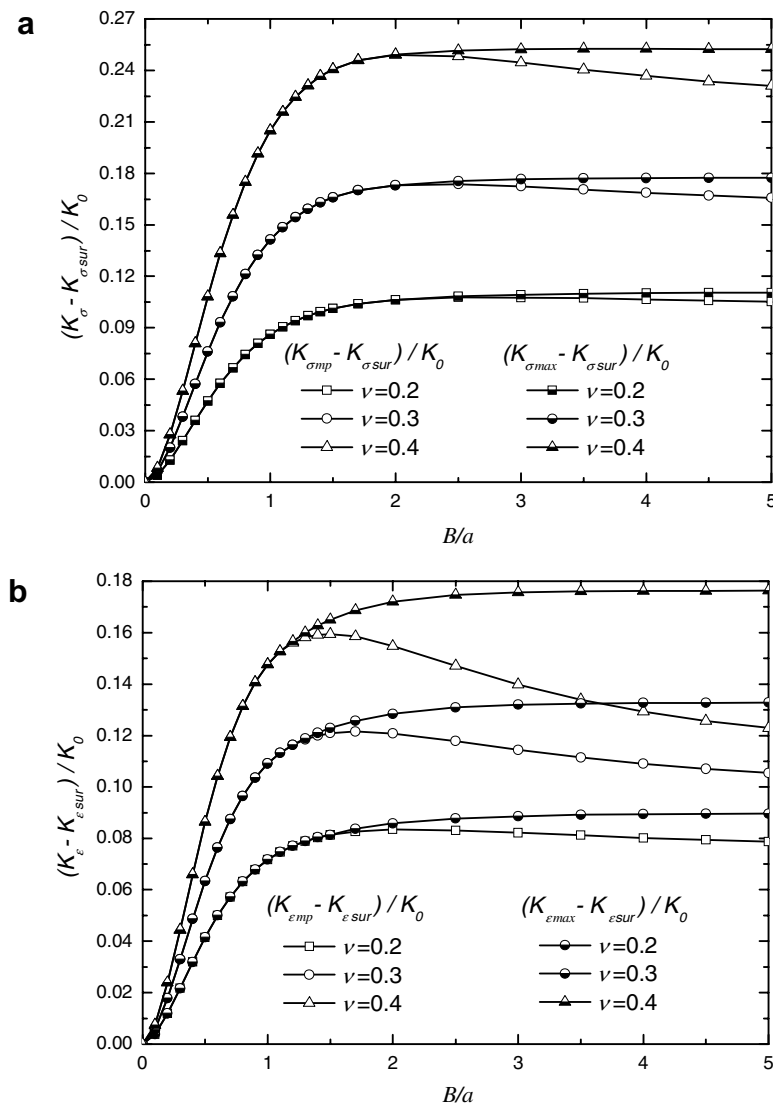


Fig. 7. The differences between mid-plan, or maximum value and surface value of (a) stress and, (b) strain concentration factor versus normalized thickness for different Poisson's ratio.

than that on the mid plane, is close to the plate surface. However, the K_σ and K_ϵ values at notch root of plate surface are the same and they are lower than that in the plane stress or plane strain states. They decrease rapidly near the plate surface and are too lower to reflect the overall stress concentration as thickness or Poisson's ratio increases. Fig. 7 also shows that it is risky to use the K_ϵ measured directly on the plate surface or calculated by 2D theories in the engineering design. The influence of thickness and Poisson's ratio on stress and strain concentration factor must be taken account of.

4.3. The relation of stress and strain concentration factor

In three-dimensional stress state, the distributions of stress and strain concentration factor are different along the thickness of notch root, shown in Figs. 4 and 6. For three-dimensional stress state of elastic medium, the strain components produced by each of the three stresses can be denoted as

$$\begin{aligned}\epsilon_{xx} &= \frac{1}{E} [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})], \\ \epsilon_{yy} &= \frac{1}{E} [\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})], \\ \epsilon_{zz} &= \frac{1}{E} [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})].\end{aligned}$$

According to Eqs. (1)–(4), the following expression can be obtained

$$K_\epsilon = K_\sigma [1 - \nu(T_x + T_z + T_x \cdot T_z)]. \quad (11)$$

It is indicated that the stress concentration factor and the strain concentration factor of the finite thickness plate are different even if the material is in elasticity state. The K_ϵ/K_σ distributions in front of notch root on different plane layer for the plate of $B/a = 4$ and $\nu = 0.3$ are shown in Fig. 8. From theory and finite element results, it is showed that the distribution of K_ϵ/K_σ depends on the Poisson's ratio and the distributions of in-plane and out-of-plane stress constraint factor. The K_ϵ/K_σ value is different in different location near the notch root. The stress concentration factor is equal to the strain concentration factor only at the notch root of plate surface, where the out-of-plane and in-plane stresses, σ_{zz} and σ_{xx} , are zero simultaneously (that is, $T_z = 0$ and $T_x = 0$), or at the notch root under the plane stress state. This has been confirmed by the FE results, shown in Fig. 5b.

The values of K_ϵ in finite thickness plate are all less than the one of K_σ except at the notch root of plate surface where the K_ϵ is equal to the K_σ . The distributions of K_ϵ/K_σ are different on different layers paralleled

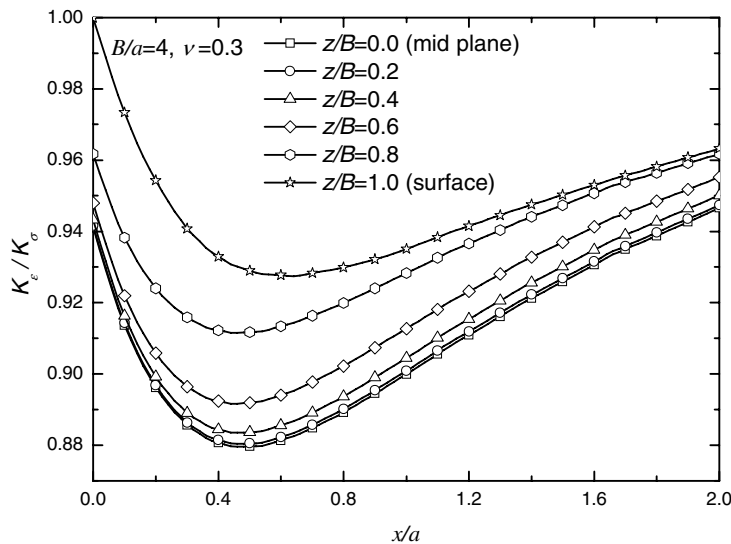


Fig. 8. The K_ϵ/K_σ distributions in front of notch root on different plane layer.

to the mid plane of plate. Each $K_e/K_\sigma - x/a$ curve in front of notch root on different layer has a minimum which quantity and location lie on the layer location, thickness and Poisson's ratio of plate, and tends to 1 when distance x tends to infinity where there is no stress concentration and the stress σ_{xx} and σ_{zz} are zero simultaneously.

The in-plane stress factor T_x is zero along the notch root. From Eq. (11), the distribution of K_e/K_σ along this line can be expressed as

$$K_e = K_\sigma[1 - \nu T_z]. \quad (12)$$

It is indicated that the stress concentration factor and the strain concentration factor of the finite thickness plate are different even along the notch root except at the notch root of plate surface, shown in Fig. 9. The distributions of K_e/K_σ along the notch root depend on the Poisson's ratio and the distributions of out-of-plane stress constraint factor. The T_z/ν distributions along the notch root depend on the thickness of plate and are insensitive to the Poisson's ratio. The out-of-plane stress constraint factor T_z is zero on the free surface of plate and its maximum value is at the notch root of mid plane. So the K_e/K_σ is 1 at the notch root of plate surface and the location of minimum K_e/K_σ value along the notch root is on the mid plane. The difference between stress and strain concentration factor at notch root of mid plane ($K_{\sigma \text{ mp}}$ and $K_{e \text{ mp}}$) increases with the out-of-plane stress constraint factor at the notch root of mid plane and Poisson's ratio. The T_z/ν at the notch root of mid plane as a function of the plate thickness for different Poisson's ratio are plotted in Fig. 10. It increases with the thickness of plate and will tend to a value. This value is in good agreement with the theoretical result $2/3$ by Sternberg et al. (1949) as ν is $1/3$ in that article. The difference between stress and strain concentration factor at the notch root of mid plane may exceed 10% as $B/a = 4$ and Poisson's ratio $\nu = 0.4$. The $K_{e \text{ mp}}/K_{\sigma \text{ mp}}$ value will tend to a constant related to Poisson's ratio, as shown in Fig. 11. The $K_{e \text{ mp}}/K_{\sigma \text{ mp}} - B/a$ curves are the monotonic descent functions of thickness. The $K_{e \text{ mp}}/K_{\sigma \text{ mp}}$ decreases from 1 at $B/a = 0$ (plane stress state) tends to each low limit value for different Poisson's ratio when B/a becomes large enough. All of the $K_{e \text{ mp}}$ value is smaller than its $K_{\sigma \text{ mp}}$ value corresponding to each thickness and Poisson's ratio. The larger the Poisson's ratio is, the higher the decreasing rate of $K_{e \text{ mp}}/K_{\sigma \text{ mp}} - B/a$ curve is and the lower the $K_{e \text{ mp}}/K_{\sigma \text{ mp}}$ tends to when B/a becomes large enough.

In engineering practice, the stress concentration factor is commonly obtained by measuring the strain on plate surface. But the stress concentration and stress concentration distributions are different on plate surface. The distributions of K_e/K_σ near the notch root of plate surface are related to the plate thickness and Poisson's ratio, shown in Fig. 12. The out-of-plane stress constraint factor is zero on the plate surface. From Eq. (11), the distribution of K_e/K_σ on the plate surface can be expressed as

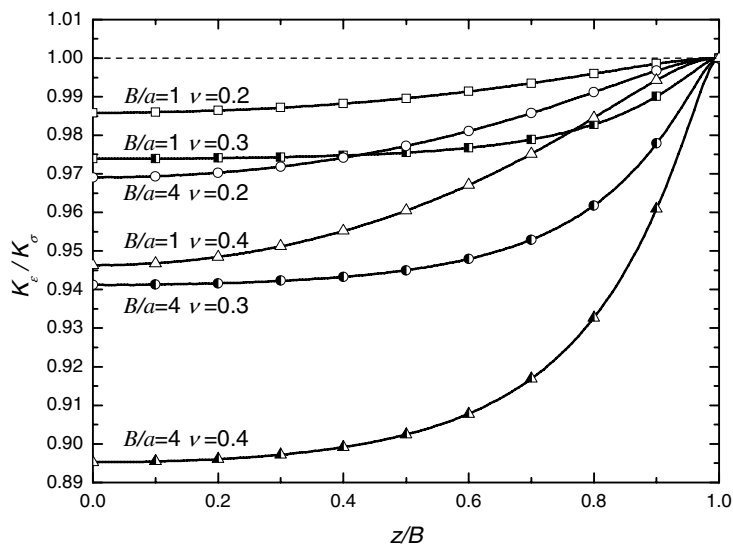


Fig. 9. The distributions of K_e/K_σ along notch root for different thickness and different Poisson's ratio.

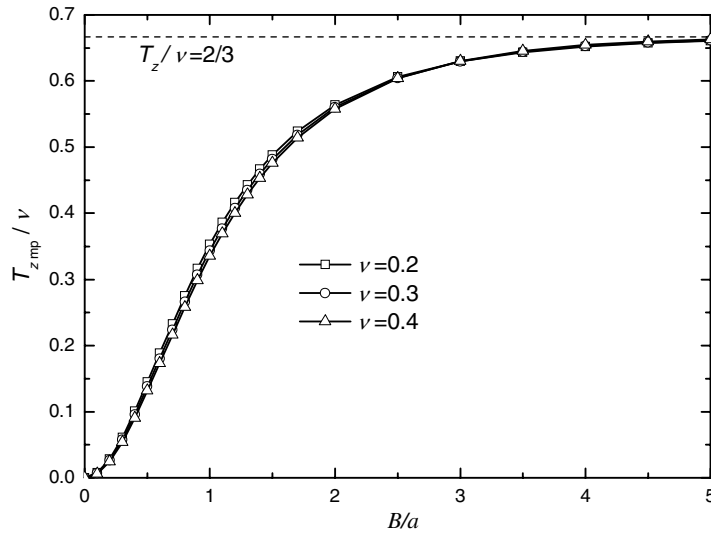


Fig. 10. The variations of out-of-plane constraint at notch root of mid plane with plate thickness for different Poisson's ratio.

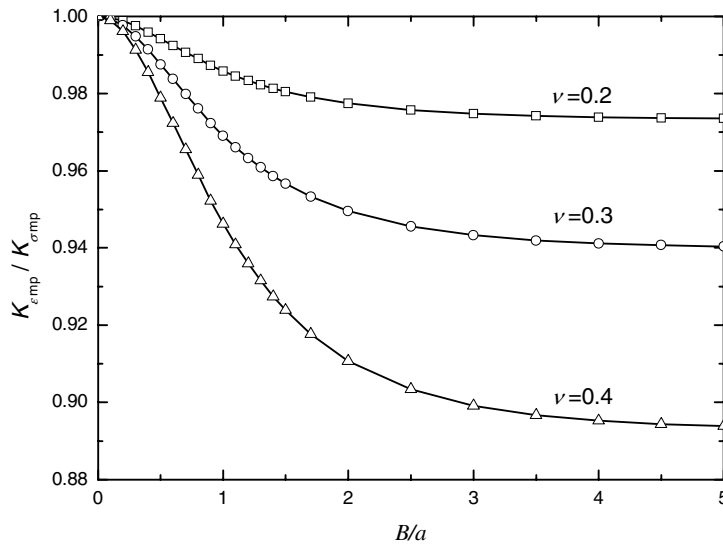


Fig. 11. The $K_{\varepsilon}/K_{\sigma}$ at the notch root of mid plane versus normalized thickness for different Poisson's ratio.

$$K_{\varepsilon} = K_{\sigma}[1 - \nu T_x]. \quad (13)$$

If we know the distribution of in-plane stress on plate surface, we can obtain the exact relation of stress concentration and strain concentration factor on it. But the distribution of strain near the notch root of finite thickness plate is very complicated. Here we substitute Eq. (9) into Eq. (13) to obtain the approximate expression of $K_{\varepsilon}/K_{\sigma} - x/a$ curve in front of notch root on plate surface,

$$\frac{K_{\varepsilon}}{K_{\sigma}} = \left[1 - \nu \frac{3\left(1 + \frac{x}{a}\right)^{-2} \left[1 - \left(1 + \frac{x}{a}\right)^{-2}\right]}{\left[2 + \left(1 + \frac{x}{a}\right)^{-2} + 3\left(1 + \frac{x}{a}\right)^{-4}\right]} \right]. \quad (14)$$

It has been confirmed in Section 3 that Eq. (9) is suitable to express the distribution of in-plane stress factor only near the notch root. So the Eq. (14) may be used to depict the distribution of $K_{\varepsilon}/K_{\sigma}$ only near the notch

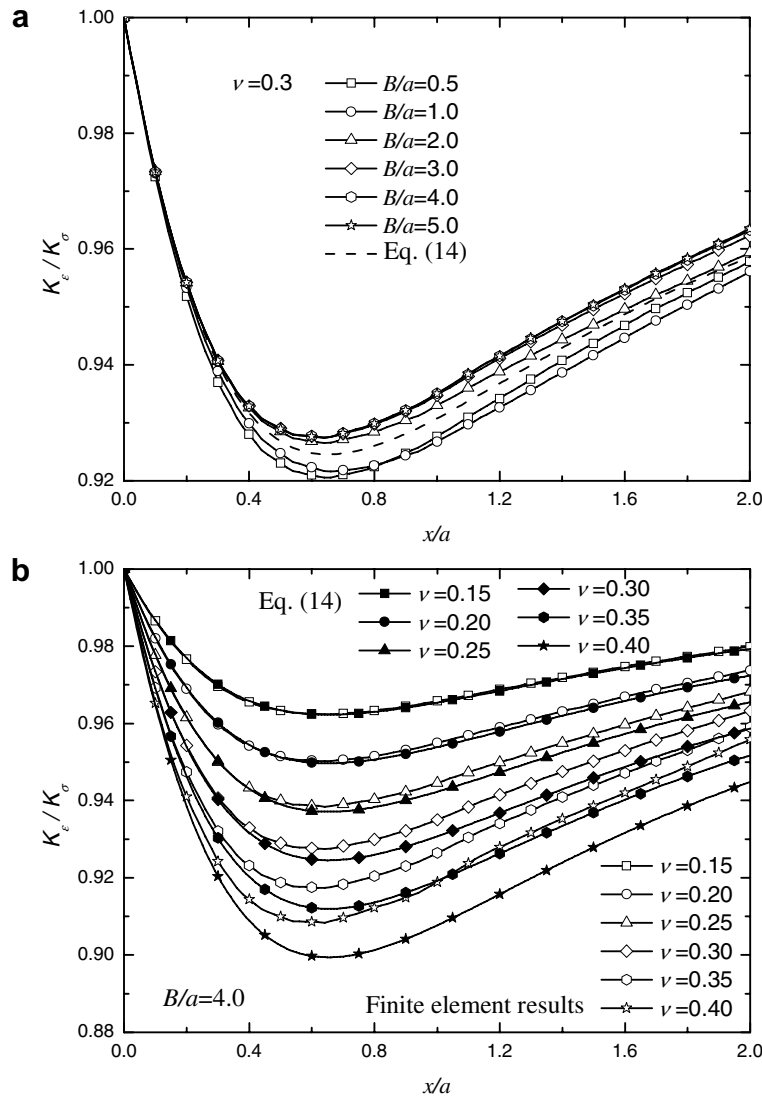


Fig. 12. The distributions of K_e/K_σ near notch root of plate surface for (a) different thickness and (b) different Poisson's ratio.

root of plate surface, shown in Fig. 12a. The distribution of K_e/K_σ on plate surface is different for the plate with different thickness or Poisson's ratio. The $K_e/K_\sigma - x/a$ curve on plate surface has a minimum which quantity lies on the plate thickness and Poisson's ratio, shown in Fig. 12b. According to Eq. (14) and 3D FE results, the location corresponding the minimum of K_e/K_σ on plate surface is sensitive to the thickness and insensitive to Poisson's ratio. It is interesting that the Eq. (14) is in good agreement with 3D FE results of $K_e/K_\sigma - x/a$ curve on plate surface for the plate with low Poisson's ratio, such as $\nu = 0.15$ and $\nu = 0.20$ of $B/a = 4.0$ plate. The difference between FE result and Eq. (14) increases and the applicable range of Eq. (14) decreases with the Poisson's ratio increasing.

4.4. Discussion of stress and strain concentration factor in finite thickness plate

From the aforementioned figures, the stress and strain concentration factors along the notch root have close relation to the thickness and Poisson's ratio of plate. They can be, respectively, expressed as functions of z/B , B/a and ν by

$$K_{\sigma}\left(\frac{z}{B}\right) = f_1\left(\frac{z}{B}, \frac{B}{a}, \nu\right), \quad (15)$$

$$K_{\varepsilon}\left(\frac{z}{B}\right) = f_2\left(\frac{z}{B}, \frac{B}{a}, \nu\right). \quad (16)$$

According to the symmetry, the stress and strain concentration factors along the notch root are of the following characters:

$$K_{\sigma}\left(\frac{z}{B}\right) = K_{\varepsilon}\left(\frac{z}{B}\right) \quad (z = \pm B).$$

For the thin plate, the maximums of $K_{\sigma}/K_{\sigma \text{ mp}} - z/B$ and $K_{\varepsilon}/K_{\varepsilon \text{ mp}} - z/B$ curves are all at the notch root of mid plane. But for the thick plate, the values of K_{σ} and K_{ε} at notch root of mid plane are the minimums of $K_{\sigma}/K_{\sigma \text{ mp}} - z/B$ and $K_{\varepsilon}/K_{\varepsilon \text{ mp}} - z/B$ curves, respectively. The point of maximum concentration factor moves away from the mid plane of plate by increasing plate thickness. Respective stress and strain concentration factors in a plate may not reach their maximums at the same location, that is, when $B/a > B_{\sigma}^*/a$

$$\frac{\partial K_{\sigma}}{\partial z} = 0 \text{ and } \frac{\partial^2 K_{\sigma}}{\partial z^2} < 0 \quad \left(\frac{z}{B} = \frac{z_{\sigma}^*}{B}\right),$$

when $B/a > B_{\varepsilon}^*/a$

$$\frac{\partial K_{\varepsilon}}{\partial z} = 0 \text{ and } \frac{\partial^2 K_{\varepsilon}}{\partial z^2} < 0 \quad \left(\frac{z}{B} = \frac{z_{\varepsilon}^*}{B}\right),$$

where z_{σ}^* and z_{ε}^* are the locations of maximum stress and strain concentration factor along the notch root, respectively. The z_{σ}^*/B and z_{ε}^*/B values of the same plate are different. But both of them increase with the plate thickness. The values of z_{σ}^*/B and z_{ε}^*/B tend to 1 when the plate thickness tends to infinite and they are zero when $B/a < B_{\sigma}^*/a$ and $B/a < B_{\varepsilon}^*/a$, respectively.

The variations of stress and strain concentration factor at the notch root of mid plane with thickness can be expressed as functions of B/a and ν by

$$\frac{K_{\sigma \text{ mp}}\left(\frac{B}{a}\right)}{K_0} = f_3\left(\frac{B}{a}, \nu\right), \quad (17)$$

$$\frac{K_{\varepsilon \text{ mp}}\left(\frac{B}{a}\right)}{K_0} = f_4\left(\frac{B}{a}, \nu\right), \quad (18)$$

respectively. The variations of maximum stress and strain concentration factor along the thickness of notch root with thickness can be expressed as functions of B/a and ν by

$$\frac{K_{\sigma \text{ max}}\left(\frac{B}{a}\right)}{K_0} = f_5\left(\frac{B}{a}, \nu\right), \quad (19)$$

$$\frac{K_{\varepsilon \text{ max}}\left(\frac{B}{a}\right)}{K_0} = f_6\left(\frac{B}{a}, \nu\right), \quad (20)$$

respectively. The variations of stress and strain concentration factor at the notch root of plate surface with thickness can be expressed by a same function of B/a and ν ,

$$\frac{K_{\text{sur}}\left(\frac{B}{a}\right)}{K_0} = f_7\left(\frac{B}{a}, \nu\right). \quad (21)$$

Here f_1 and f_2 are the functions of z/B , B/a and ν . f_3, f_4, f_5, f_6 and f_7 are the functions of B/a and ν , respectively. These functions are of the following characters:

$$f_3\left(\frac{B}{a}, \nu\right) = f_4\left(\frac{B}{a}, \nu\right) = f_5\left(\frac{B}{a}, \nu\right) = f_6\left(\frac{B}{a}, \nu\right) = f_7\left(\frac{B}{a}, \nu\right) = 1, \quad \text{when } \frac{B}{a} = 0.$$

When $\frac{B}{a} \leq \frac{B_{\sigma}^*}{a}$,

$$\frac{K_{\sigma \text{mp}}(\frac{B}{a})}{K_0} = \frac{K_{\sigma \text{max}}(\frac{B}{a})}{K_0}, \text{ so } f_3\left(\frac{B}{a}, v\right) = f_5\left(\frac{B}{a}, v\right).$$

When $\frac{B}{a} > \frac{B_\sigma^*}{a}$,

$$1 \leq f_3\left(\frac{B}{a}, v\right) < f_5\left(\frac{B}{a}, v\right).$$

When $\frac{B}{a} \leq \frac{B_\sigma^*}{a}$,

$$\frac{K_{\sigma \text{mp}}(\frac{B}{a})}{K_0} = \frac{K_{\sigma \text{max}}(\frac{B}{a})}{K_0}, \text{ so } f_4\left(\frac{B}{a}, v\right) = f_6\left(\frac{B}{a}, v\right).$$

When $\frac{B}{a} > \frac{B_\epsilon^*}{a}$,

$$f_4\left(\frac{B}{a}, v\right) < f_6\left(\frac{B}{a}, v\right).$$

Here B_σ^* and B_ϵ^* are the transition thickness of stress and strain concentration factor, respectively.

When $\frac{B}{a} \rightarrow \infty$,

$$\frac{K_{\sigma \text{mp}}(\frac{B}{a})}{K_0} = f_3\left(\frac{B}{a}, v\right) \rightarrow 1.$$

As $\frac{B}{a} \rightarrow \infty$, $T_z \rightarrow \frac{2v}{3}$. According to the relation of stress and strain concentration factor, Eq. (12),

$$\frac{K_{\sigma \text{mp}}(\frac{B}{a})}{K_0} = f_4\left(\frac{B}{a}, v\right) \rightarrow 1 - \frac{2}{3}v^2.$$

$$\frac{K_{\sigma \text{max}}(\frac{B}{a})}{K_0} = f_5\left(\frac{B}{a}, v\right) \rightarrow C_5(v) > 1,$$

$$\frac{K_{\epsilon \text{max}}(\frac{B}{a})}{K_0} = f_6\left(\frac{B}{a}, v\right) \rightarrow C_6(v) < 1,$$

$$\frac{K_{\text{sur}}(\frac{B}{a})}{K_0} = f_7\left(\frac{B}{a}, v\right) \rightarrow C_7(v) < 1.$$

Here, $C_5 > C_6 > C_7$ for the plate with same Poisson's ratio. C_5 , C_6 , and C_7 are the constants related to Poisson's ratio. According to the FE results, the values of C_5 , C_6 , and C_7 are about 1.024, 0.979 and 0.846, respectively, for the plate of Poisson's ratio $v = 0.3$.

5. The stress and strain concentration factor distributions near plate surface

The through-thickness distribution of out-of-plane strain normalized by each value of plate surface is shown in Fig. 13. The plate thickness is normalized by hole radius in this figure. The out-of-plane strain distributions of different thickness plate are very similar near the plate surface. The influence of surface on strain and stress field is only in a limited thickness near the surface. But within this narrow thickness, the variety of deformation is very intense and the intense degree is almost same for different thickness plate. For the thin plate, the influence thickness of surface may exceed the half thickness of plate and the deformation is influenced by the plate surface of opposite side. The thicker the plate is, the weaker the opposite side influences the deformation. For the thick plates, the distributions of out-of-plane strain are almost same near the surface. It's evident Kane-Mindlin's plate theory and similar theories assuming constant strain through the thickness can only be applied to the small central zone of thick plate with stress concentration problem. It may not be reasonable to use this kind of assumption near the plate surface.

Because of symmetry, the tangent line of $K_\sigma/K_{\sigma \text{sur}} \sim z/a$ curve at mid plane is a horizontal line. The maximum stress concentration factor is not always at the mid plane. The maximum value and its location depend on the plate thickness, shown in Fig. 14. The thinner the plate is, the more the $K_\sigma/K_{\sigma \text{sur}} \sim z/a$ curve is linear. For

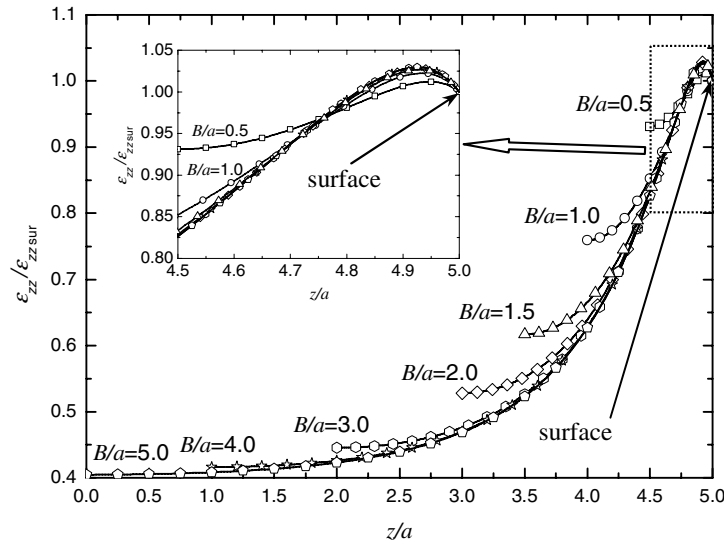


Fig. 13. The through-thickness distributions of out-of-plane strain normalized by each value of plate surface.

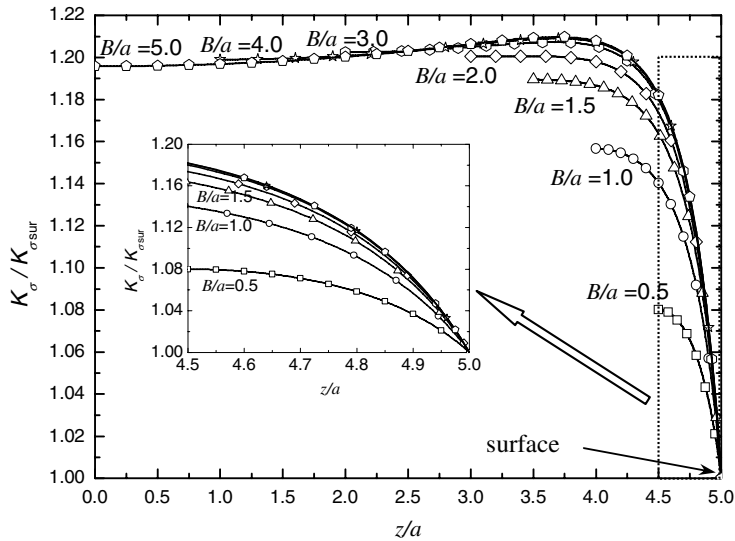


Fig. 14. The through-thickness distributions of stress concentration factor normalized by each value of plate surface.

the thin plate, the maximum stress concentration factor is at the notch root of mid plane, the maximum stress concentration factor and the curvature of $K_\sigma/K_{\sigma\text{sur}} \sim z/a$ curve increase as the plate thickness increases. The influence of surface on stress concentration factor distribution increases with the plate thickness increasing. But the influence thickness of surface is a limited thickness near surface. The stress concentration factor in the influence thickness will increase and tend to a constant value, $C_5 K_0$, with the plate thickness increasing. For the thick plate, the stress concentration factor at the notch root of mid plane will tend to the value of plane stress state, which is equal to the value of plane stress state, when the plate thickness becomes large enough. So there is a maximum point of the stress concentration factor near the plate surface, where the value of stress concentration factor is larger than the value of mid plane and plate surface. From the FE results, it is found that the locations of maximum are almost equidistance from surface for the plate with different thickness that exceeds the transition thickness of stress concentration factor. This distance is about $1.2a$ from the surface. For the thick plate, there is a plain segment in $K_\sigma/K_{\sigma\text{sur}} \sim z/a$ curve near the mid plane. The thicker the plate is, the longer the plain segment in $K_\sigma/K_{\sigma\text{sur}} \sim z/a$ curve is and the closer the $K_\sigma/K_{\sigma\text{sur}} \sim z/a$ curves are to each other

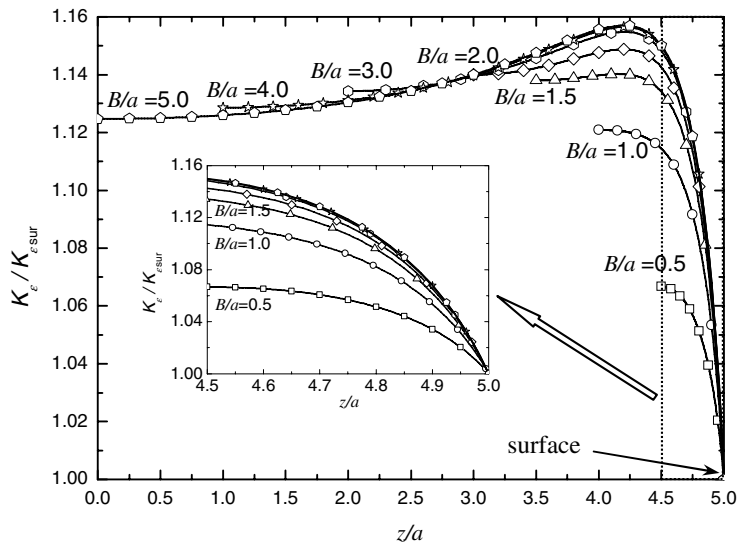


Fig. 15. The through-thickness distributions of strain concentration factor normalized by each value of plate surface.

near surface. The $K_e/K_{e,sur} \sim z/a$ curve is similar to the $K_\sigma/K_{\sigma,sur} \sim z/a$ curve, shown in Fig. 15. The strain concentration factor in the influence thickness will increase and tend to a constant value, $C_6 K_0$, with the plate thickness increasing. For the thick plate, the strain concentration factor at the notch root of mid plane will tend to $(1 - 2\nu^2/3)K_0$, when the plate thickness becomes large enough. There is a maximum point of the strain concentration factor near the plate surface. The distance from maximum point to surface is about $0.7a$ for the plate with different thickness that exceeds the transition thickness of strain concentration factor.

In summary, the influence of plate surface on stress field is only in a limited thickness near the surface, but the variety of deformation near the notch root is very intense in this narrow thickness. The strain and stress distributions of different thickness plate are very similar near surface. They are influenced by the couple effect of surface and internal three-dimensional constraint. The plate surface influences on strain and stress field of different thickness plate are almost same, whereas the influences of internal three-dimensional constraints are dependent on the plate thickness. This couple effect in thick plate makes the stress and strain near plate surface higher than that near mid plane and on the plate surface. But in the thin plate, the internal three-dimensional constraint is weak and the strain and stress distributions are mainly influenced by the surface. The maximum stress and strain are at the notch root of mid plane.

6. Conclusions

In the present article, the elastic stress and strain fields of finite thickness large plate containing a hole subjected to uniaxial tension are systematically examined using 3D finite element method. The sensitivity of the stress and strain concentration factor to plate thickness as well as the Poisson's ratio was examined. By comparison of 2D fields, some special characters of 3D fields are revealed:

- (1) The maximum stress and strain concentration factors do not always occur on the mid plane of the plate. They occur on the mid plane only in thin plates. The location of maximum concentration factor moves away from the mid plane of plate by increasing plate thickness. The locations of maximum stress and strain concentration factor are not same in the same thick plate.
- (2) The stress concentration factor and the strain concentration factor of the finite thickness plate are different even if the plate is in elasticity state. The stress concentration factor is equal to the strain concentration factor only at the notch root of plate surface. The stress and strain concentration factors at notch root of mid plane, maximum point and plate surface are the functions of thickness, which depend on the Poisson's ratio of plate.

- (3) The difference between maximum and surface value of stress concentration factor is a monotonic ascent function of thickness, which depends on the Poisson's ratio of plate. The larger the thickness or Poisson's ratio of plate is, the larger this difference is. This difference relationship of strain concentration factor is similar to the one of stress concentration factor. But the difference magnitude of stress concentration factor is larger than the one of strain concentration factor in the same plate.
- (4) The strain and stress distributions of different thickness plate are very similar near plate surface. They are influenced by the couple effect of surface and internal three-dimensional constraint. The plate surface influences on strain and stress field of different thickness plate are almost same in a limited thickness near the surface, whereas the internal three-dimensional constraints of plate are dependent on the plate thickness. This couple effect in thick plate makes the stress and strain concentration factor near plate surface higher than that near mid plane.

Acknowledgement

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